DEPARTMENT OF THE ARMY

CORPS OF ENGINEERS

BEACH EROSION BOARD

OFFICE OF THE CHIEF OF ENGINEERS

ACCURACY OF HYDROGRAPHIC SURVEYING IN AND NEAR THE SURF ZONE

TECHNICAL MEMORANDUM NO. 32

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MARCH 1953

FOREWORD

During the period June 1949 to April 1951, a Field Research Group of the Beach Erosion Board made repetitive measurements of beach profiles, wave characteristics, suspended sediment and beach and bottom materials, and recorded certain meteorologic data in the vicinity of Mission Bay, San Diego County, California. Mr. Donald R. Forrest was in field charge of the group, assisted by Mr. Robert L. Harris who was responsible for surveys, instrument installation, and operation and maintenance of equipment.

The following report by Messrs. Saville and Caldwell is the first of several expected to be produced from the field data obtained at Mission Bay. Repetitive measurements of beach profiles are frequently used for quantitative determination of volumetric changes, and heretofore there has been no reliable basis for assessing the probable error resulting from such measurements. Although the results presented cannot be universally applied without considering the need for a correction factor applicable to local conditions, it is expected that they will provide a needed aid in planning and evaluating beach surveys.

The major part of this report was presented at the Third Conference on Coastal Engineering, held in Boston in November 1952, and is expected to be published as part of the proceedings of the conference. It is also being published at this time as a Technical Memorandum of the Beach Erosion Board because of its obvious application to beach erosion studies and the consequent advantages of its inclusion in the Board's report series. The opinions expressed therein are not necessarily those of the Beach Erosion Board.

This report is published under authority of Public Law 166, 79th Congress, approved July 31, 1945.

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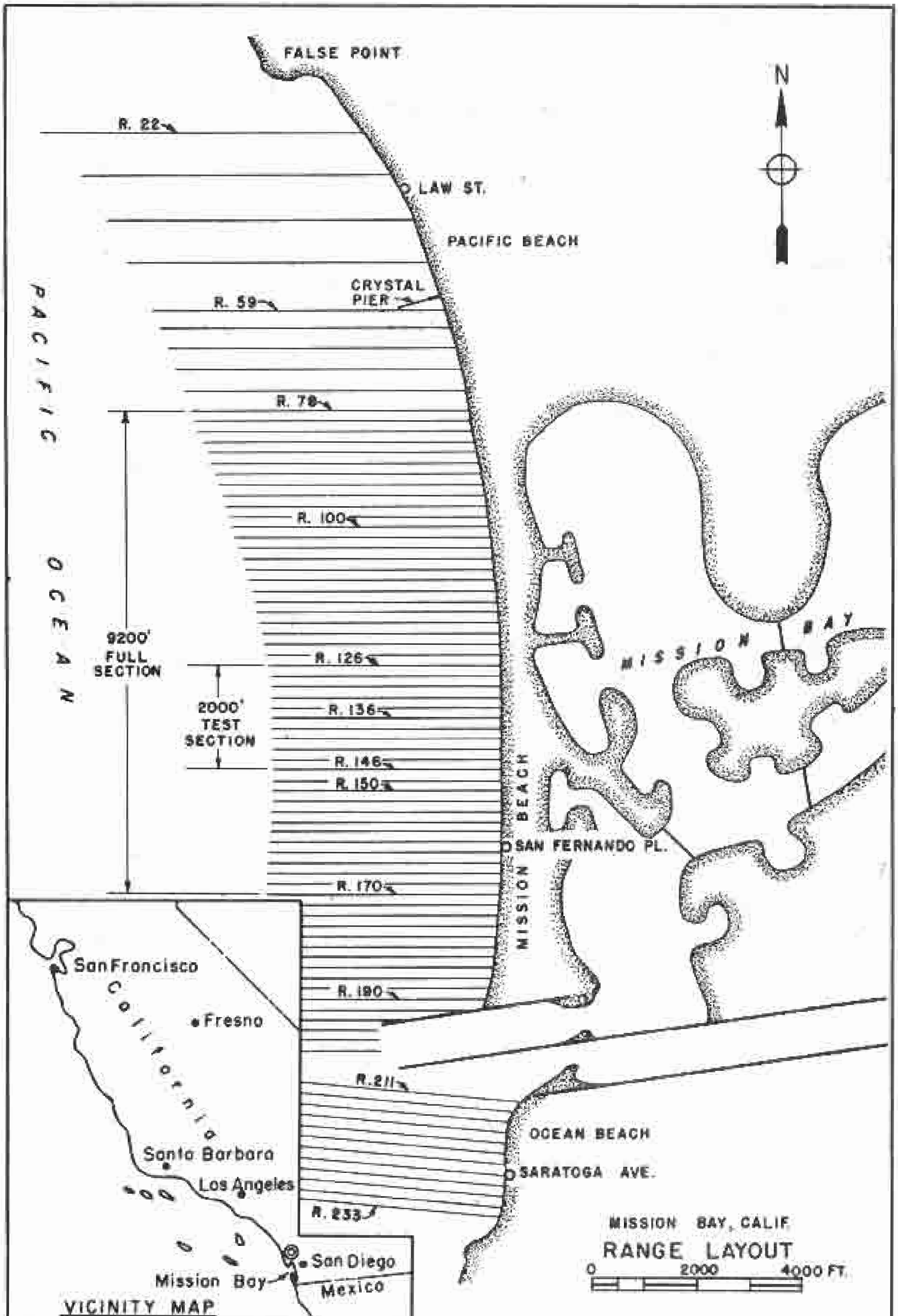

FIG. 1 - LOCALITY MAP

ACCURACY OF HYDROGRAPHIC SURVEYING IN AND NEAR THE SURF ZONE*

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INTRODUCTION

The analysis and solution of most beach erosion problems are based to a significant degree on the quantitative changes in the bottom hydrography as observed in successive surveys. Critical decisions as to the dominant direction of littoral drift, the average rate of this drift, and the onshore-offshore movement of material are based largely on such hydrographic surveys. As the net changes between successive surveys are usually small compared to the area being studied, the degree of accuracy or comparability of the hydrographic surveys is of considerable importance. For instance, a net change of 100,000 cubic yards over one square mile of beach represents an average change in depth of only about 0.1 foot. Thus, it can be seen that uncompensated errors in depth measurement of as little as 0.1 foot can produce indications of significant littoral sand movement which might not exist in reality.

The errors involved in hydrographic work may be attributed almost entirely to two different causes. The first of these, a sounding error, results from errors inherent in the sounder and the methods involved in reducing the sounder data to an actual bottom profile (i. e. tide corrections, elimination of the effect of waves, water termperature corrections, etc.). The second, a spacing error, results from the fact that a particular profile may not be entirely representative of its assigned section of beach.

The sounding error is a measure of the accuracy (or inaccuracy) with which the profile deduced from the sounder record actually represents the bottom hydrography along the particular range being sounded; as such it may be determined as a function of the reproducibility of this profile by the repetition of a series of soundings. The spacing error is a measure of the accuracy (or inaccuracy) with which the particular profile portrays the characteristics of the contiguous beach area; as such it may be determined as a function of the reproducibility of the hydrography of a beach area by using various spacings between adjacent profiles.

It was the purpose of this study to determine on a statistical basis the degree of accuracy that could be expected in hydrographic survey work where comparability of successive surveys is a prime consideration. Tests to determine the magnitude of these two types of error were made at Mission Beach, California, (Figure 1). Mission Beach is a relatively long,

*The major part of this paper was presented at the Third Conference of Coastal Engineering in Boston in November 1952, and is being published in the Proceedings of that meeting.

straight beach, with essentially parallel contours, and no radical changes of bottom hydrography along its length, and as such, is representative of many of the southern California beaches. The results of these tests may be expected to apply to other beaches of the same type.

The tests were made under normal operating conditions by the Field Research Group of the Beach Erosion Board; i.e., standard Beach Erosion Board procedures were used in checking the tide, the sounding instruments, and the position of the survey boat so that the results could be considered applicable to actual hydrographic surveys made by the Field Group. A description of the standard survey techniques used by the Field Research Group is given in The Bulletin of the Beach Erosion Board, July 1947.

DETERMINATION OF SOUNDING ERROR

Description of Tests. The test to determine sounding error involved the repeated sounding of a single profile eight times successively in a 5-hour period. The survey extended from the shore line to the -50-foot mean lower low water contour on Beach Erosion Board profile range 136 at Mission Beach. This range is about 5500 feet north of the Mission Bay jetties and the -50-foot contour is about 4250 feet offshore. The range was established by the Field Research Group in connection with other work in the area. The test was made on 3 November 1950 while swells of about two feet in height were running. The tide variation was 0.4 foot during the 5-hour period; corrections of the sounding records were made for this variation. An amphibious truck, DUKW, was used as the floating equipment for the survey. In making the tests, a Bludworth NK-2 echo-sounder was used while the DUKW was floating; a leadline was used while the wheels of the DUKW were grounded in traversing the shallow water section of the profile.

Analysis of Echo-Sounder Data. The echo-sounder, or sonic, data and the leadline soundings were analyzed separately. The echo-sounder charts were first corrected for tide elevations and the soundings taken off at 250-foot intervals starting at a point 750 feet from the base line. The tabulation of results is shown on Table 1. This table shows the corrected soundings for the eight test runs and covers the area from about the -6-foot to the -50-foot mean lower low water contour, a distance of about 3,500 feet. The table also shows an average profile column obtained by averaging the eight separate profiles.

As with most statistical data, there are several ways of effecting an analysis. However, only two methods appeared to have enough engineering significance in the present case to warrant a set of calculations. The first method assumes that the average profile is the correct profile for the 5-hour period and then studies the deviation of each of the eight profiles from the average. The second method assumes that the deviation of one profile from the succeeding profile is a better measurement of the degree of accuracy with which successive surveys can be compared. The data have been analyzed in both ways.

The deviation of the individual soundings from the average sounding for the comparable station is shown in Table 2. The deviations for each profile are summarized algebraically on the table; each summation is in turn divided by the number of stations, 15, in order to establish the average deviation, d, of the profile from the average profile. This average deviation is a measure of the error that would be introduced in a set of computations by using a single profile instead of the average profile; thus Run 3 gives a profile for the echo-sounder portion of the record which averages 0.130 foot below the average profile. These average profile deviations, d, can be handled collectively by the statistical formula

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

where σ is the standard deviation and n is the number of observations. The result is

$$\sigma = \sqrt{\frac{0.08524}{8}} = 0.103 \text{ foot}$$

The probable error, P.E., in any one profile is given by

$$\text{P.E.} = 0.6745\sigma = 0.069 \text{ foot.} \quad (\text{say } 0.07 \text{ foot})$$

This indicates that any one profile obtained by the echo sounder can be expected to have an uncompensated error averaging 0.07 foot.

The second method of analysis involves comparing each profile with the succeeding profile. In this manner, no attempt is made to establish the absolute profile as was done with the average profile in the preceding paragraph; rather the comparison is on the basis of the comparability of successive profiles. The statistical analysis based on this reasoning is given in Table 3. In this case it can be seen that the profile of Run 1 is compared to Run 2, then Run 2 to Run 3, and so on. Finally, Run 8 is compared back to Run 1. The summation and statistical handling is the same as used previously and shows for the echo-sounder portion of the record a standard deviation, σ, of 0.118 foot, and a probable error of 0.08 foot. It is to be noted that the probable error indicated by this analysis is of the same order as for the first analysis (0.08 foot against 0.07 foot). Attention is also called to the fact that the deviation for the comparison of Run 8 to Run 1 was well below the average deviation, indicating that there was no systematically increasing error over the 5-hour test period.

In considering this indication of an 0.07 to 0.08-foot uncompensated error it should be kept in mind that this figure is probably an optimistic one due to the fact that the comparative profiles were taken on the same day with the same personnel and equipment and with a relatively small tide variation. These factors would tend to make the error somewhat less than would be the case if the surveys were taken several weeks or months apart. Also, any constant error that might have been effective on the day of the soundings, such as in the instruments, the submergence of the sounder, or the tide adjustment, is not included in the 0.07-foot figure.

Analysis of Leadline Data. A leadline was used for sounding whenever the wheels of the DUKW were grounded. Table 1 shows the leadline soundings as well as the sonic soundings taken during the running of the eight test profiles. These soundings were analyzed statistically in the same manner as the echo sounder records and it was found that:

(a) A comparison of profile deviation against the "average" profile showed an uncompensated probable error of 0.11 foot.

(b) A comparison of successive profiles showed an uncompensated probable error of 0.20 foot.

It is seen that these probable errors with the leadline are considerably greater than the probable errors for that portion of the profile sounded by echo sounder. However, the portion of the profile covered by leadline is generally a minor portion of the entire profile so that the quantitative error is usually not as great in the overall picture. In the Mission Bay tests, about 4,000 feet of profile were sounded by echo sounder and about 300 feet by leadline.

The fact that the actual beach profile for the eight test runs was probably slightly different for each run is appreciated. However, this does not change the analysis given above, as no hydrographic survey is made simultaneously over all profiles. Instead the profiles are run successively as in the test and the test runs would appear to indicate the degree of comparability of the profiles, which was the purpose of the test.

Of some significance in considering the results of the analysis given above is the fact that the portable echo-sounders used in most beach profile work are rated as having an accuracy of $\pm \frac{1}{2}$ foot at a 50-foot depth. It should be noted that the sounder accuracy is expressed in feet at 50 feet and not as a percentage; this is done because some of the errors in the sounder vary with depth whereas others are independent of depth. Thus the error could be expected to be less at 10 feet than at 50 feet but not as much less as the ratio of depths might indicate. The fact that during the eight test runs discussed above the same echo-sounder was used by the same crew and the entire test covered only a 5-hour period would tend to hold the sounder error to a minimum. The usual bar checks were made to adjust the sounder before starting the tests.

Application to a Survey Consisting of More Than One Profile. The preceding discussion applies to the sounding error to be expected over a single profile. Most hydrographic surveys involve the use of a number of profiles to determine the hydrography of a given area. The use of multiple profiles makes it likely that the uncompensated errors in one profile will be somewhat compensated by a similar error opposite in sign on another profile. The eight profiles used in the preceding discussion were accordingly analyzed toward the end of discovering the sounding error to be expected in the use of multiple profiles.

In making this analysis, the eight profiles of Table 1 were compared to the average profile shown in the same table. The eight profiles were compared individually to the average and the resultant deviations compared statistically; the results of this comparison have already been discussed and are shown on Table 2. The results indicated for the sonic-sounder portion a standard deviation of 0.103 foot based on the use of a single profile on which to establish a comparison.

The indicated errors for every possible combination of two profiles were then averaged. The results established a standard deviation for the offshore portion of 0.0676 foot based on the use of two profiles. The comparison was continued for all possible combinations of three, four, five, six, seven, and eight profiles with the results shown in Table 4. In using these results, two factors must be kept in mind:

(1) That the results should not be construed as indicating to what degree the profiles are representative of the section of beach which they are assumed to represent. The present portion of this report is pointed toward indicating the surveying errors; the degree to which a selected profile may be considered representative will be discussed later in this report.

(2) That the entire set of computations is influenced by the fact that only eight profiles were used and that these eight were averaged to give the reference or base profile. This condition affects the lower end of the curve much more than the upper end; for instance Table 4 indicates a zero deviation if eight profiles were used, which is obviously unrealistic. However, it is believed that the figures for the use of one or two profiles are not too greatly influenced by the fact that only eight profiles were used as a basis for the computations.

If the value based on the use of a single profile is assumed to be correct, then values for the use of any number of profiles may be derived from error theory to give

$$\sigma_n = \frac{\sigma_1}{\sqrt{n}}$$

where σ_n represents the standard deviation to be expected from the use of n profiles; and σ_1 is the standard deviation for a single profile. σ_1 was previously shown to be 0.103 foot for the sonic portion of the profile and 0.199 for the leadline portion. Values for the probable error may be derived similarly, and

$$P.E._n = \frac{P.E._1}{\sqrt{n}}$$

Values for the standard deviation and probable error computed by this formula are also shown in Table 4. Figure 2 shows the variation of the sounding error as computed by error theory if it is assumed that the value for a single profile is correctly obtained from the average of the eight test profiles. Also shown are the points obtained from using all the possible combinations of the test profiles for the sonic portion of the test. As may be seen the points obtained for the combination of two and three profiles do not differ greatly from the error theory curve, and this supports strongly the assumption that the value for the single profile is very nearly correct.

The data from Figure 2 have the dimensions of feet, and can be expressed as cubic feet per lineal foot of shore per foot of profile and hence can be reduced to a relationship of probable cubage error per foot of shore as related to the number of profiles utilized in the survey under consideration. A tabulation of this relationship for the sonic sounder, as computed from Figure 2, is given in Table 5, and for leadline soundings in Table 6. The relationships for both leadline and sonic portions are shown as a series of curves in Figure 3. The values given in Tables 5 and 6 or Figure 3 are readily applied to the analysis of the probable surveying error inherent to a given survey of a beach. Knowing the number of profiles used, and the average length of these profiles, the cubage error per foot of beach can be computed. The product of this unit error and the length of beach gives the probable cubage error over the study area. It should be kept in mind that the cubage errors indicated in Tables 5 and 6 are <u>per linear foot of beach.</u> As an example, for a 10,000-foot section of beach, surveyed by 20 profiles each 4,000 feet long, the total probable sounding error would be (0.57) (4) (10,000) = 22,800 cubic yards.

From the above it can be seen that surveying errors may enter the analysis of a beach problem to a significant degree if too few profile lines are used in the study. It should again be emphasized that these errors represent sounding error alone and take no account of a spacing error.

It should be noted that the computations discussed above and tabulated in Tables 2 and 4 were based on the use of fifteen soundings for the sonic sounder section of each profile. The question arises as to the effect on the comparative accuracy of the profile line of increasing the number of soundings. This effect was investigated by taking the same eight profiles previously used and picking off soundings at 125-foot intervals instead of 250-foot intervals; this resulted in thirty soundings for comparison, or double the number originally used. An intercomparison of these eight profiles with thirty soundings each was then worked out on the same basis as described above. Table 7 shows a comparison of the results using 30 soundings per profile with the results using 15 sounding per profile; the very close agreement in the results indicates that the use of 15 soundings per line was sufficient to establish the accuracy characteristics of the profile and that nothing

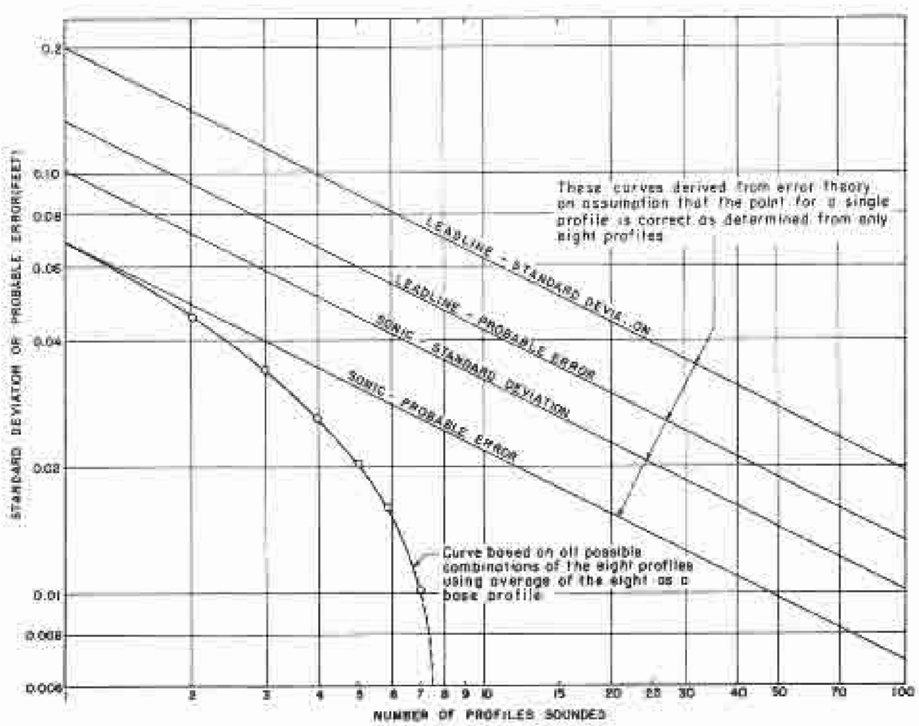


FIG. 2 - VARIATION OF SOUNDING ERROR WITH NUMBER OF PROFILES

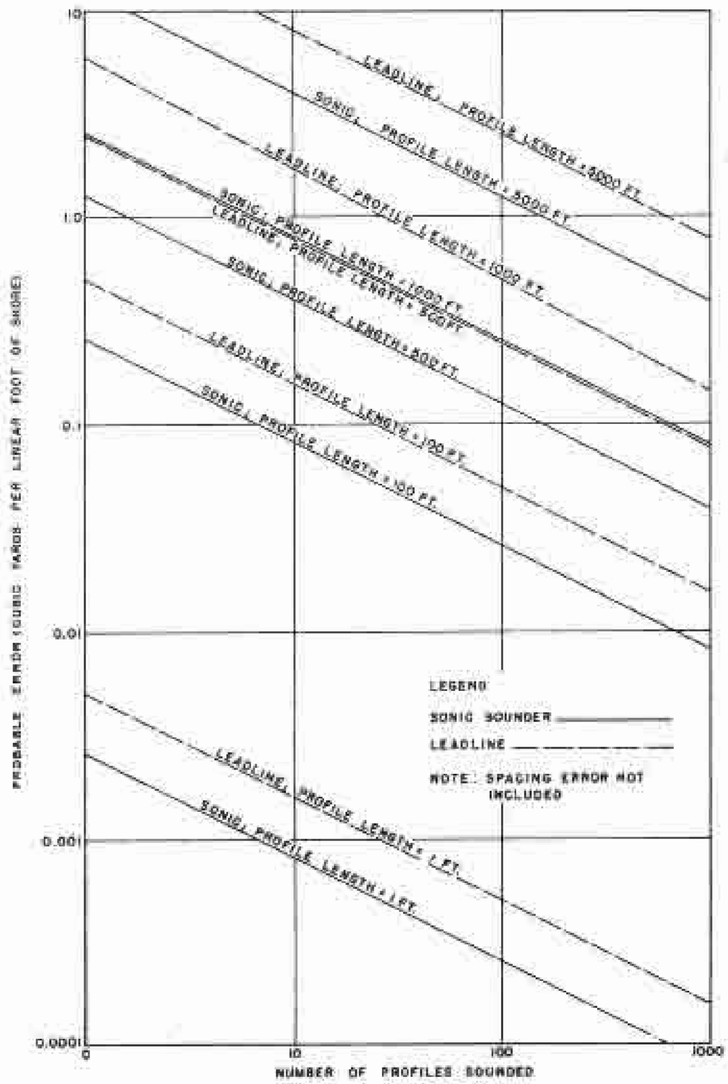


FIG. 3 - RELATION OF PROBABLE SOUNDING ERROR TO NUMBER AND LENGTH OF PROFILES

would be gained by increasing the number of soundings utilized in the comparison.

DETERMINATION OF SPACING ERROR

Description of Test. As stated in the introduction, the spacing error is considered as the error resulting from the fact that a particular profile may not be entirely representative of its assigned section of beach. The tests to determine spacing error involved the use of data obtained from two different sets of surveys. These were:

(a) The sounding at Mission Beach of a 2,000-foot test section consisting of eleven ranges spaced 200 feet apart at approximately one week intervals between 12 May and 8 September 1950. In addition, three surveys were made in April 1951, making a total of nineteen surveys. The ranges involved were established by the Field Research Group of the Beach Erosion Board in connection with other work, and were designated Beach Erosion Board ranges 126-146. The mid-range of the section was about 5,500 feet north of the Mission Bay jetties and the -50-foot contour is about 4,250 feet offshore. All surveys extended from the shore line to the -50-foot mean lower low water contour.

(b) The sounding at Mission Beach of a 9,200-foot section of beach consisting of 47 ranges spaced 200 feet apart at approximately three-month intervals between June 1949 and April 1951. A total of eight surveys were involved. Again, all surveys extended to the -50-foot mean lower low water contour. The ranges involved were Beach Erosion Board ranges 78-170; range 170 is about 2,100 feet north of the Mission Bay jetties; range 78 is slightly over two miles north of the jetties, and about 2,000 feet south of Crystal Pier.

The entire beach in the Mission Beach area is sand and has essentially straight and parallel contours, with no radical changes in underwater hydrography along its length; this uniformity of the beach was considered desirable for this study as the profiles might reasonably be expected to be representative of an extensive section of beach.

Analysis of Echo-Sounder Data. The echo-sounder data and the leadline soundings were analyzed separately. The echo-sounder charts were corrected for tide elevation, and, as in the analysis for sounding error, soundings were taken off at 250-foot intervals along each range starting from a point 750 feet from the baseline. A tabulation of the soundings of the eleven profiles for the 2,000-foot test section for the survey of 12 May 1950 is shown in Table 8, as is an average profile obtained by averaging the eleven separate profiles. The deviation of any particular profile from this average profile is a measure of the error involved if only that profile were used to determine the hydrography of the area. Similarly, the error involved in using any particular set of profiles to indicate this hydrography may be measured as the sum of the deviations of the profiles from the average profile, if these deviations are weighted according to the area which each profile is assumed to represent. For the 12 May 1950 survey of the 2,000-foot test section, a tabulation of the deviation of each sounding and the overall deviation of each range from the average profile is shown in Table 9. Similar tabulations were made for each of the nineteen surveys of the 2,000-foot test section and each of the eight surveys of the 9,200-foot section. Figure 4 shows, for the sonic portion, a typical average profile, and also the average deviation of each individual profile from this average profile.

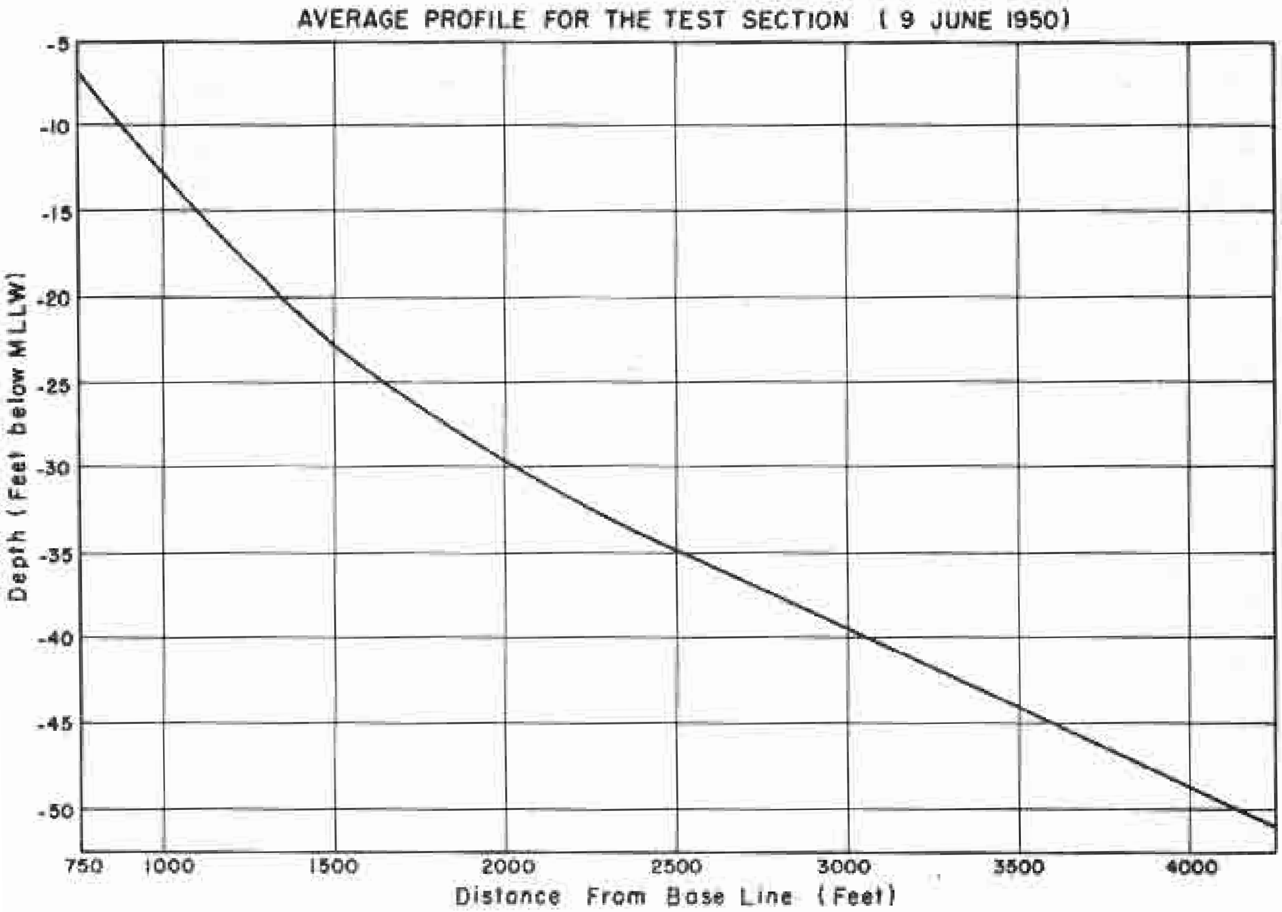
The error involved in using a number of different combinations of profiles rather than the full number of profiles was determined for each survey. The combined error for a series of evenly spaced profiles was determined as the algebraic sum of the deviations of each individual profile from the average profile determined from full survey data. This gave a variation of profile spacing of 400 to 2,000 feet for the test section and 400 to 9,200 feet for the full section. A tabulation of these errors (for the combinations of profiles selected) for the test section surveys is shown in Table 10, and for the full section surveys in Table 11. The nineteen different values (one for each survey) involved in the test section and the eight different values involved in the full survey may be analyzed statistically to obtain a standard deviation and a probable error by the formulas used in the preceding section. These values are also shown in Tables 10 and 11.

Several of the combinations of profile lines used have the same spacing, and these may be combined to give a single value of the standard deviation for each spacing. For example, in the test section, using a combination of ranges 3 and 9 gives a 1,000-foot spacing, as does also the combination of ranges 1, 6, and 11. The former results in a probable error of 0.072 foot and the latter in one of 0.053 foot. These may be combined by taking the square root of the sum of the squares to give a single, more accurate value of 0.064 foot for the probable error. This combining has been done for both the test section and the full survey, and values of standard deviation and probable error for the various spacings are shonein Table 14. The values for the probable error have been plotted in Figure 6, and a curve drawn to fit the points. The scatter is surprisingly small, and it is thought that the curve represents fairly accurately the error which may be expected due to profile spacing on beaches having a hydrography generally similar to that of Mission Beach and sounded by sonic methods.

Due to the large number of surveys and profiles used, the sounding error (discussed previously) is negligible (each point plotted represents the results from the combination of a minimum of 24 profiles, and most points are obtained from several hundred profiles) -- and hence the error determined by this method may be attributed entirely to spacing error. This type of error is of greater magnitude than the sounding error, and may reach considerable values if the spacing is large.

That portion of the error curve for spacings between 100 and 2,500 feet may be represented very closely by the linear function

$$P.E. = 0.013 + 4.84s \times 10^{-5}$$



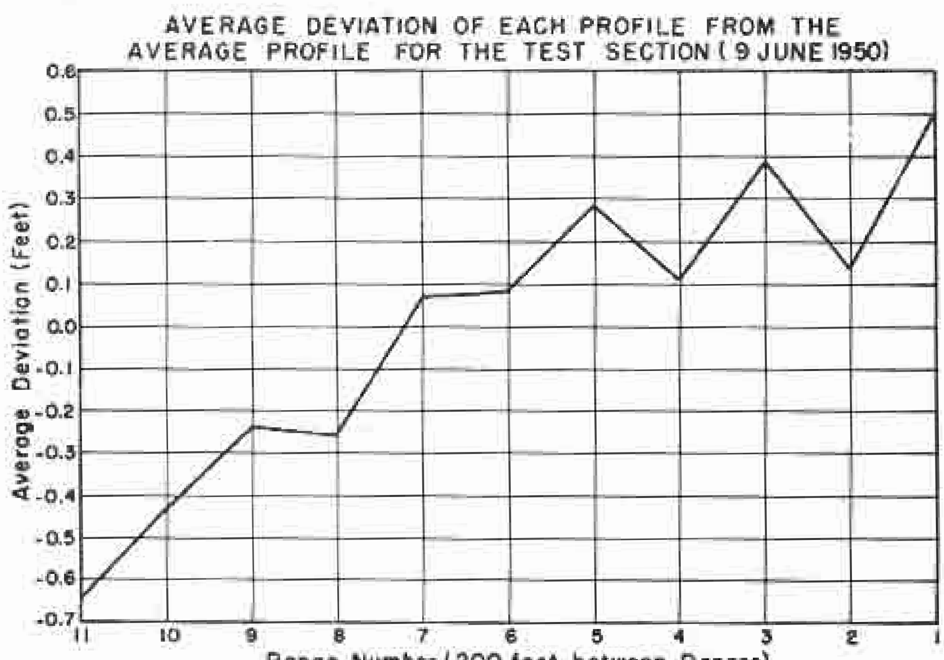


FIG. 4 - AVERAGE PROFILE AND DEVIATION - SONIC DATA

where s is the spacing in feet.

It was suspected that if the data used were too meager, the spacing error might decrease somewhat as the number of profiles at that spacing was increased — the spacing error between one set of profiles tending to compensate somewhat for the spacing error between the next set of profiles. If this were true, then the points obtained from the 9,200-foot section, having many more profiles, should lie somewhat beneath the points determined from the 2,000-foot test section. Such is not significantly the case however, and it is thought that the curve is an accurate portrayal of the spacing error.

Analysis of Leadline Data. A similar analysis was performed on the leadline data, and values for each profile of the 12 May 1950 survey of the test section are shown in Tables 8 and 9, along with the sonic data. A typical average profile for the leadline portion, and the deviation therefrom are shown in Figure 5. The errors involved in using different combinations of profiles rather than the full number of profiles are tabulated in Tables 12 and 13 (similar to Tables 10 and 11 for the sonic data). Where the combinations of profile lines used have the same spacing, the errors have been combined, in the same way as the sonic data, to give a single average error for each spacing. This has been done for both the test section and the full section, and values of standard deviation and probable error are shown in Table 14. The values for the probable error have also been plotted in Figure 6, where they may be compared with the points determined from the sonic data. A curve of best fit has been drawn.

As may be seen from the figure, there is considerably more scatter in the leadline data than in the sonic data, and the points determined from the leadline data generally lie above (show greater error than) those from the sonic data. Since both curves refer to spacing error alone, the method of sounding should not affect the error, and the curves should be identical. The difference observed between the curves may be attributed to the different depths involved, i.e., the fact that the inshore, shallower portion of the beach (where the leadline data were taken) is much less regular than the offshore portion, and a particular profile there would be expected to be much less representative of the surrounding area than it would farther offshore where the hydrography is more regular.

It is to be noted that the curves of best fit cross each other at a spacing of 6,000 feet. This seems completely illogical, and it is thought that enough data were probably not obtained to determine accurately the errors for the 9,200-foot spacing. Twenty-four profiles were used to determine these points (as opposed to 56 for the 4,600-foot spacing, and more for the lesser spacings), and, as may be seen from Tables 11 and 13, a rather large spread in these points is observed. It is thought that the curve for the shallower water (from the leadline data) should continually lie above that for the deeper water (sonic data) and the dashed lines in the figure indicate what are thought to be the more probable extensions of the curves. Actually this is somewhat of an academic question, as the large errors involved for spacings of this magnitude practically

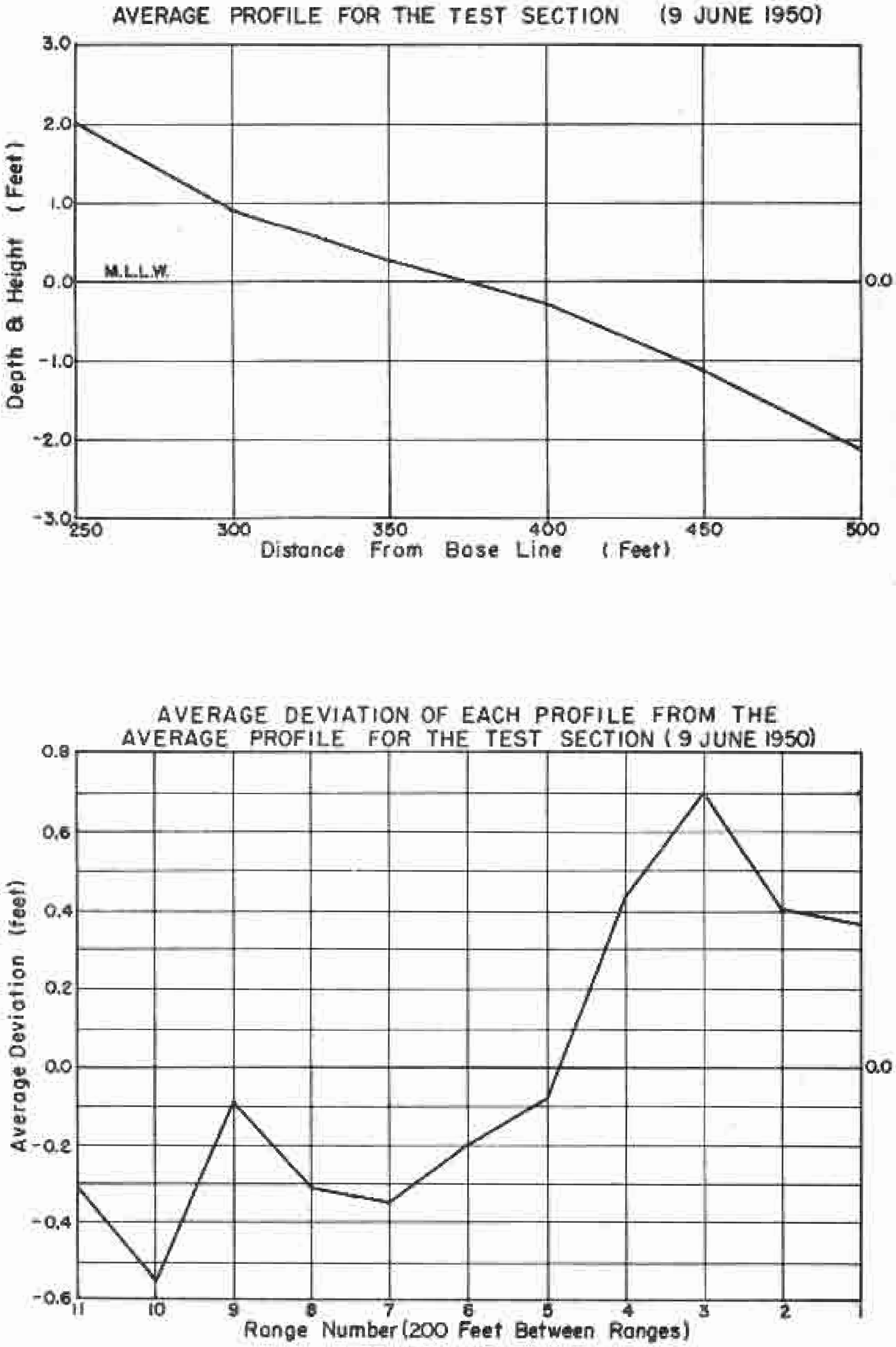


FIG. 5 - AVERAGE PROFILE AND DEVIATION-LEADLINE DATA

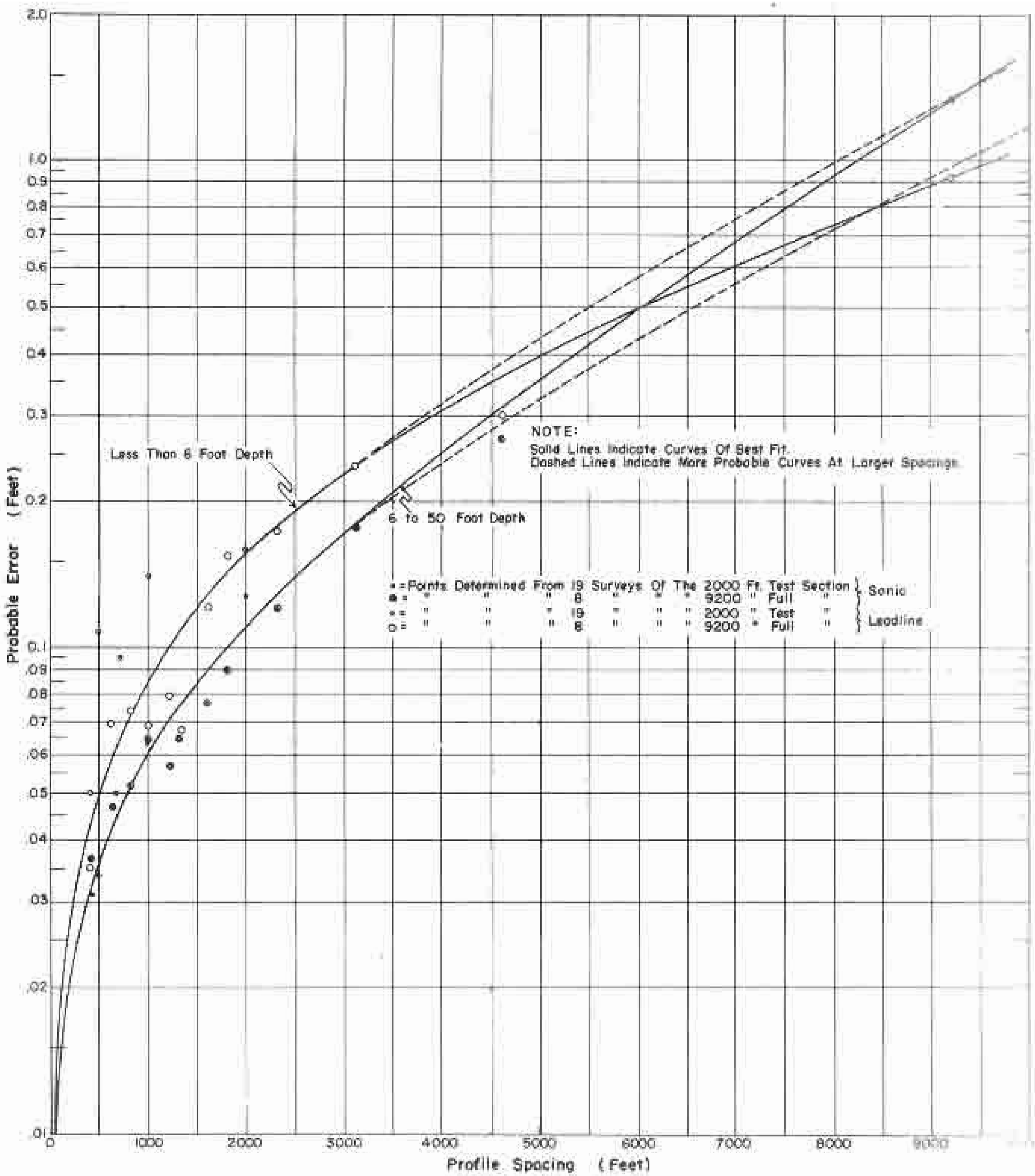


FIG. 6 - PROBABLE SPACING ERROR

preclude their use in the field for obtaining quantitative data.

As with the sonic data, the error for the leadline data for shallow water use can also be expressed quite accurately as a linear function between spacings of about 100 and 2,500 feet. This is

$$P.E. = 0.016 + 7.1s \times 10^{-5}$$

where the spacing, s, is in feet.

APPLICATION TO AN ACTUAL SURVEY

The total error to be expected in any particular survey will be a combination of the sounding error and the spacing error, and may be determined, for beaches similar in hydrography to Mission Beach, from the curves shown herein. If e denotes the total probable error, e_s the probable spacing error, and e_a the sounding error, then

$$e = \sqrt{e_a^2 + e_s^2}$$

and the probable error, E, in cubic yards is

$$E = \frac{\sqrt{e_a^2 + e_s^2}\, LL'}{27}$$

where L is the length of the beach in feet and L' the length of the profile in feet.

Examples of this combined error are shown in Figures 7a (for the deeper water sounded by echo sounder) and 7b (for the shallower water sounded by leadline). Values of probable error are shown as feet for general application and also as cubic yards for the specific cases of a 10,000-foot stretch of beach with sonically sounded 4,000-foot profiles in depths of 6 to 50 feet or with leadline sounded 500-foot profiles in depths less than 6 feet. If the portion in deeper water is also sounded by leadline, a similar set of curves can be simply drawn in the same manner, using the spacing error as determined for deeper water by sonic methods, and the sounding error as determined from leadline data.

In an actual survey, if E_s denotes the error to be expected in shallower water, and E_d that to be expected in the deeper portions, the total probable error, E_T, is the sum of these or

$$E_T = E_s + E_d$$

A specific example for a 10,000-foot stretch of beach with 4,500-foot profiles, where the shoremost, shallow water section of 500 feet was sounded by leadline, and the deeper, seaward 4,000 feet was sonically sounded, has been worked out and is shown in Figure 8.

As may be readily seen from any of these figures, the probable spacing

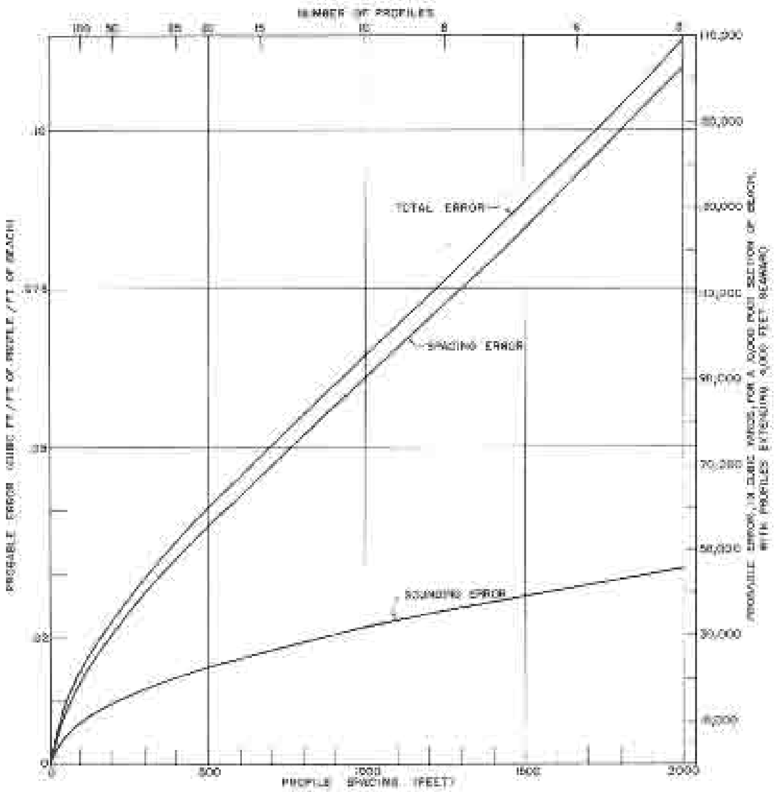


FIG 7a - PROBABLE ERRORS IN VOLUME DETERMINATIONS
SURVEY BY ECHO SOUNDER - DEPTHS 6 TO 50 FEET

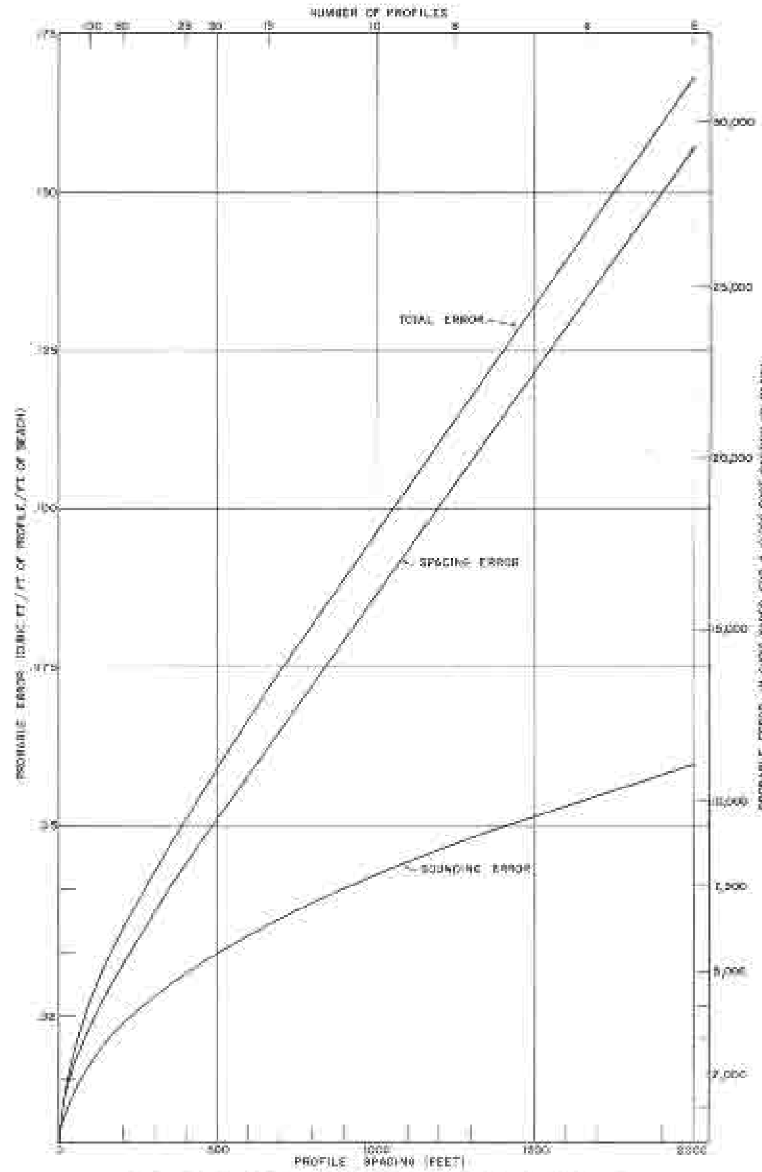


FIG. 7b - PROBABLE ERRORS IN VOLUME DETERMINATIONS
SURVEY BY LEADLINE - DEPTHS LESS THAN 6 FEET

error for a large spacing reaches a considerable cubage. It is interesting to note that, at least for these particular cases, although the sounding error is quite appreciable, it is so small in comparison to the spacing error that it has only a relatively small effect on the total error. It may also be noted that for many cases the shallow water portion of the profile is so short compared to the entire profile that very nearly as accurate an estimate of probable error is obtained by using the data for deeper water alone. For example, for the case shown in Figure 8, values of total probable error for several spacings have been computed by using 500 feet of shallow water profile and 4,000 feet of deep water profile, and also by assuming that the entire profile could be represented by 4,500 feet of deep water profile. The comparisons are shown in Table 15 below.

TABLE 15

PROBABLE ERROR (CUBIC YARDS)

Spacing (feet)	500 feet shallow 4,000 feet deep	4,500 feet deep	Difference %
1000	113,800	108,000	5.1
500	70,900	67,400	4.9
200	39,800	37,800	5.0

As may be seen, the difference between the two cases is small (about 5 percent), and it is thought that in many cases probable errors can be adequately determined by applying the errors for the deeper water portion to the entire profile.

The analysis of sounding and spacing errors presented in this report appears to demonstrate that the cubage errors — due to the facts that profiles of a hydrographic survey are not strictly comparable either among themselves or to a previous survey (sounding error), and that any particular profile does not necessarily represent accurately the bottom area which it is assumed to describe (spacing error) — can introduce serious misinterpretations as to the rate and direction of movement of littoral drift. For instance, in the Mission Beach area, for a 10,000-foot stretch of beach, it is seen that for a very small range spacing (200 feet) an error of 40,000 cubic yards can still be more or less expected in the cubage computations; while for the relatively large spacing of 1,000 feet, an error of 114,000 yards can be expected. In many beach studies errors of these magnitudes could produce completely misleading interpretations of the test data. It is therefore recommended that the presence of such errors be considered as a distinct possibility in the interpretation of test data based on the comparison of successive hydrographic surveys.

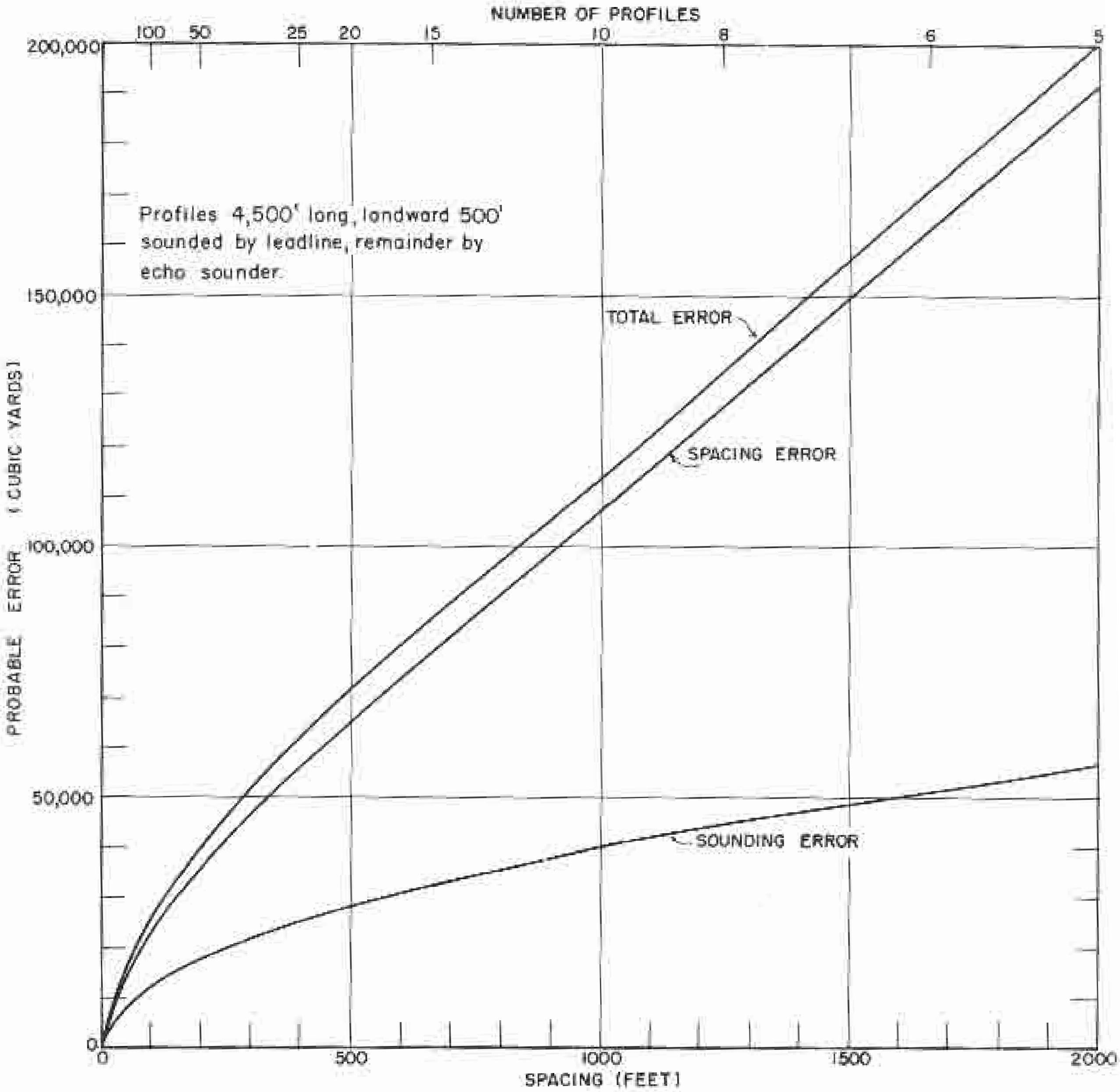


FIG. 8 - PROBABLE VOLUME ERRORS FOR 10,000-FOOT BEACH

APPENDIX

TABLE 1

Soundings Taken at Range 136, Mission Bay, California
3 November 1950

Distance from Base Line (ft.)	Soundings in feet MLLW for Run Number 1	2	3	4	5	6	7	8	Average for all runs
				Sonic Soundings					
750	-7.4	-7.5	-7.5	-7.4	-7.3	-7.6	-7.7	-7.5	-7.49
1000	14.3	14.2	14.2	14.5	14.2	14.1	14.2	14.0	14.21
1250	19.1	18.9	19.1	19.4	18.9	18.7	18.9	18.8	18.98
1500	23.6	23.4	23.8	23.6	23.3	23.7	23.4	23.7	23.56
1750	26.9	27.3	27.4	27.6	26.9	27.1	27.1	27.5	27.22
2000	30.0	30.4	30.3	30.3	30.3	30.3	30.1	30.3	30.25
2250	32.7	33.0	32.9	33.2	33.0	32.9	32.5	32.8	32.88
2500	35.4	35.4	35.2	35.6	35.2	35.3	35.0	35.3	35.30
2750	37.7	37.4	37.6	37.9	37.6	37.4	37.5	37.5	37.57
3000	39.8	39.8	40.1	39.9	39.7	39.6	39.8	39.7	39.80
3250	41.8	41.8	42.3	42.4	41.8	41.7	41.8	41.9	41.94
3500	44.1	44.1	44.7	44.5	44.1	44.1	43.8	44.3	44.21
3750	46.6	46.5	46.8	46.5	46.5	46.5	46.4	46.5	46.54
4000	48.9	48.8	48.8	48.7	48.7	48.6	48.5	48.6	48.70
4250	50.9	51.2	50.8	50.9	50.8	50.8	50.8	51.0	50.90
				Leadline Soundings					
250	2.1	1.8	1.5	2.4	1.8	2.4	2.5	1.9	2.05
300	1.1	1.4	1.4	2.2	1.0	1.1	1.4	1.5	1.39
350	0.9	0.5	0.6	0.8	+0.3	+0.8	+0.8	0.6	0.66
400	+0.3	+0.3	+0.2	+0.4	-0.4	-0.3	+0.5	+0.3	+0.16
450	-0.7	-0.7	-0.5	-0.6	-1.1	-1.4	-0.5	-0.6	-0.76
500	-1.5	-1.5	-1.5	-2.0	-2.3	-2.0	-1.7	-1.5	-1.75

Note: Soundings were taken over a 5-hour period and have been corrected for tide.

TABLE 2

Deviation (in feet) of Actual Profiles from Average Profile

Distance from Base Line (ft.)	Profile being compared to average profile 1	2	3	4	5	6	7	8
				Sonic Soundings				
750	+.09	-.01	-.01	+.09	+.19	-.11	-.21	-.01
1000	-.09	+.01	+.01	-.29	+.01	+.11	+.01	+.21
1250	-.12	+.08	-.12	-.42	+.08	+.28	+.08	+.18
1500	-.04	+.16	-.24	-.04	+.26	-.14	+.16	-.14
1750	+.32	-.08	-.18	-.38	+.32	+.12	+.12	-.28
2000	+.25	-.15	-.05	-.05	-.05	-.05	+.15	-.05
2250	+.18	-.12	-.02	-.32	-.12	-.02	+.38	+.08
2500	-.10	-.10	+.10	-.30	+.10	0	+.30	0
2750	-.13	+.17	-.03	r.33	-.03	+.17	+.07	+.07
3000	- 0	0	-.30	-.10	+.10	+.20	0	+.10
3250	+.14	+.14	-.36	-.46	+.14	+.24	+.14	+.04
3500	+.11	+.11	-.49	-.29	+.11	+.11	+.41	-.09
3750	-.06	+.04	-.26	+.04	+.04	+.04	+.14	+.04
4000	-.20	-.10	-.10	0	0	+.10	+.20	+.10
4250	0	-.30	+.10	0	+.10	+.10	+.10	-.10
Total d	-0.35	-0.15	-1.95	-2.85	+1.25	+1.15	+2.05	+0.15
Ave. d	-.023	-.010	-0.130	-0.190	+0.083	+0.077	+0.137	+0.010
d^2	.000545	.0001	.0169	.0361	.006944	.005878	.018678	.0001

$\Sigma d^2 = 0.085245 \quad n = 8 \quad \sigma = \sqrt{\frac{0.085245}{8}} = \sqrt{0.010656} = 0.103$ ft.

Probable error (sonic soundings) = (0.6745) (0.103) = 0.069 ft.

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Leadline Soundings

	1	2	3	4	5	6	7	8
250	+.05	-.25	-.55	+.35	-.25	+.35	+.45	-.15
300	-.29	+.01	+.01	+.81	-.39	-.29	+.01	+.11
350	+.24	-.16	-.06	+.14	-.36	+.14	+.14	-.06
400	+.14	+.14	+.04	+.24	-.56	-.46	+.34	+.14
450	+.06	+.06	+.26	+.16	-.34	-.64	+.26	+.16
500	+.25	+.25	+.25	-.25	-.55	-.25	+.05	+.25
Total d	+.45	+.05	-.05	+1.45	-2.45	-1.15	+1.25	+.45
Ave. d	+.075	+.008	-.008	+0.242	-0.408	-0.192	+0.208	+.075
d^2	.005625	.000069	.000069	.058403	.166736	.036736	.043403	.005625

$\Sigma d^2 = 0.316666 \quad n = 8 \quad \sigma = \sqrt{0.316666/8} = \sqrt{0.03953} = 0.199$ ft.

Probable error (leadline soundings) = (0.6745) (0.199) = 0.134 ft.

TABLE 3

Deviation (in feet) of Each Profile from the Succeeding Profile

Distance from Base Line in ft.	Profiles being compared 1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-1
				Sonic Soundings				
750	+.1	0	-.1	-.1	+.3	+.1	-.2	-.1
1000	-.1	0	+.3	-.3	-.1	+.1	-.2	+.3
1250	-.2	+.2	+.3	-.5	-.2	+.2	-.1	+.3
1500	-.2	+.4	-.2	-.3	+.4	-.3	+.3	-.1
1750	+.4	+.1	+.2	-.7	+.2	0	+.4	-.6
2000	+.4	-.1	0	0	0	-.2	+.2	-.3
2250	+.3	-.1	+.3	-.2	-.1	-.4	+.3	-.1
2500	0	-.2	+.4	-.4	+.1	-.3	+.3	+.1
2750	-.3	+.2	+.3	-.3	-.2	+.1	0	+.2
3000	0	+.3	-.2	-.2	-.1	+.2	-.1	+.1
3250	0	+.5	+.1	-.6	-.1	+.1	+.1	-.1
3500	0	+.6	-.2	-.4	0	-.3	+.5	-.2
3750	-.1	+.3	-.3	0	0	-.1	+.1	+.1
4000	-.1	0	-.1	0	-.1	-.1	+.1	+.3
4250	+.3	-.4	+.1	-.1	0	0	+.2	-.1
Total d	+.5	+1.8	+0.9	-4.1	+0.1	-0.9	+1.9	-0.2
Ave. d	+0.033	+0.12	+0.06	-0.273	+0.007	-0.06	+0.127	-0.013
d^2	0.0011	0.0144	0.0036	0.0747	0.0000	0.0036	0.0160	0.0001

$\Sigma d^2 = 0.1135 \quad n = 8 \quad \sigma = \sqrt{\frac{0.1135}{8}} = \sqrt{0.0142} = 0.119$ foot.

Probable error = (0.6745) (0.119) = 0.080 foot.

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Distance from Base Line in ft.	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-1
				Leadline Soundings				
250	+.3	+.3	-.9	+.6	-.6	-.1	+.6	-.2
300	-.3	0	-.8	+1.2	-.1	-.3	-.1	+.4
350	+.4	-.1	-.2	+.5	-.5	0	+.2	-.3
400	0	+.1	-.2	+.8	-.1	-.8	+.2	0
450	0	-.2	+.1	+.5	+.3	-.9	+.1	+.1
500	0	0	+.5	+.3	-.3	-.3	-.2	0
Total d	+.4	+.1	-1.5	+3.9	-1.3	-2.4	+0.8	0
Ave. d	+.067	+.017	-0.25	+0.65	-0.217	-0.4	+0.133	0
d	0.0044	0.0003	0.0625	0.4225	0.0469	0.16	0.0177	0

$\Sigma d^2 = 0.7143 \quad n = 8 \quad \sigma = \sqrt{\frac{0.7143}{8}} = \sqrt{0.893} = 0.299$ foot

Probable error = (0.6745) (0.299) = 0.201 foot.

TABLE 4

Study of relation of number of profiles used to the average accuracy of the profiles

Number of profiles used at a time	Number of combinations possible and used	Computed on basis of average for eight profiles: Standard deviation (feet)	Computed on basis of average for eight profiles: Probable error (feet)	Error Theory: Standard deviation (feet)	Error Theory: Probable error (feet)
(For offshore sections sounded by echo-sounder)					
1	8	0.103	0.069	0.103	0.069
2	28	0.068	0.046	0.072	0.049
3	56	0.050	0.034	0.059	0.040
4	70	0.039	0.026	0.051	0.035
5	56	0.030	0.020	0.046	0.031
6	28	0.023	0.016	0.042	0.028
7	8	0.015	0.010	0.039	0.026
8	1	0	0	0.036	0.024
(For inshore section sounded by leadline)					
1	8	0.199	0.134	0.199	0.134
2	28	0.130	0.088	0.141	0.095
3	56	0.097	0.065	0.115	0.077
4	70	0.075	0.051	0.099	0.067
5	56	0.058	0.039	0.089	0.060
6	28	0.043	0.029	0.081	0.055
7	8	0.028	0.019	0.075	0.051
8	1	0	0	0.070	0.047

TABLE 5
Probable Sounding Errors in Beach Surveys Made with Echo Sounder

Number of Profiles used	Standard deviation (feet)	Probable error (feet)	Probable error in cu. yd. per linear foot of shore, when using profiles with an average length of — 1 ft.	100 ft.	500 ft.	1000 ft.	5000 ft.
1	0.103	0.069	0.00255	0.255	1.27	2.55	12.7
2	0.072	0.049	0.00182	0.182	0.91	1.82	9.1
3	0.059	0.040	0.00148	0.148	0.74	1.48	7.4
4	0.0510	0.0342	0.00127	0.127	0.63	1.27	6.3
5	0.0457	0.0308	0.00114	0.114	0.57	1.14	5.7
6	0.0418	0.0280	0.00104	0.104	0.52	1.04	5.2
8	0.0361	0.0243	0.00090	0.090	0.45	0.90	4.5
10	0.0321	0.0217	0.00080	0.080	0.40	0.80	4.0
15	0.0264	0.0177	0.00066	0.066	0.33	0.66	3.3
20	0.0229	0.0154	0.00057	0.057	0.29	0.57	2.9
30	0.0186	0.0126	0.00047	0.047	0.23	0.47	2.3
40	0.0161	0.0110	0.00041	0.041	0.20	0.41	2.0
50	0.0145	0.0097	0.00036	0.036	0.18	0.36	1.8
75	0.0118	0.0079	0.00029	0.029	0.15	0.29	1.5
100	0.0102	0.0069	0.00026	0.026	0.13	0.26	1.3
150	0.0084	0.0056	0.00021	0.021	0.10	0.21	1.0
200	0.0072	0.0049	0.00018	0.018	0.09	0.18	0.9
500	0.0046	0.0031	0.00011	0.011	0.06	0.11	0.6
1000	0.0032	0.0022	0.00008	0.008	0.04	0.08	0.4

TABLE 6

Probable Sounding Errors in Beach Surveys Made by Leadline

Number of Profiles used	Standard deviation (feet)	Probable error (feet)	Probable error in cu. yd. per linear foot of shore, when using profiles with an average length of 1 ft.	100 ft.	500 ft.	1000 ft.	5000 ft.
1	0.199	0.134	0.00496	0.496	2.48	4.96	24.8
2	0.141	0.094	0.00348	0.348	1.74	3.48	17.4
3	0.115	0.076	0.00282	0.282	1.41	2.82	14.1
4	0.099	0.068	0.00252	0.252	1.26	2.52	12.6
5	0.088	0.059	0.00218	0.218	1.09	2.18	10.9
6	0.081	0.054	0.00200	0.200	1.00	2.00	10.0
8	0.070	0.047	0.00174	0.174	0.87	1.74	8.7
10	0.063	0.042	0.00156	0.156	0.78	1.56	7.8
15	0.051	0.034	0.00126	0.126	0.63	1.26	6.3
20	0.044	0.030	0.00111	0.111	0.56	1.11	5.6
30	0.036	0.025	0.00093	0.093	0.46	0.93	4.6
40	0.031	0.021	0.00078	0.078	0.39	0.78	3.9
50	0.027	0.019	0.00070	0.070	0.35	0.70	3.5
75	0.023	0.016	0.00059	0.059	0.30	0.59	3.0
100	0.020	0.013	0.00050	0.050	0.25	0.50	2.5
150	0.016	0.011	0.00041	0.041	0.20	0.41	2.0
200	0.014	0.009	0.00035	0.035	0.17	0.35	1.7
500	0.009	0.006	0.00022	0.022	0.11	0.22	1.1
1000	0.006	0.004	0.00011	0.016	0.08	0.16	0.8

TABLE 7

Study of Effect of Number of Soundings per Profile on the Average Accuracy of a Profile

Number of Profiles used at a time	Standard deviation* in feet using 15 soundings per profile	30 soundings per profile
1	0.103	0.103
2	0.0676	0.0675
3	0.0504	0.0503
5	0.0302	0.0302
6	0.0225	0.0224
7	0.0147	0.0147

* In computing these deviations, the various profiles and combinations were compared to the average profile of the eight profiles as was done in Tables 2 and 4. When succeeding profiles were compared in the manner done in Table 3, the use of 30 soundings per profile showed a standard deviation of 0.0118 foot which is identical with the results shown in Table 3 for 15 soundings per profile.

TABLE 8
SOUNDINGS TAKEN ON TEST SECTION AT MISSION BAY, CALIFORNIA
12 May 1950

SOUNDINGS IN FEET MLLW FOR RANGE NUMBER

Distance from Base Line (ft.)	(1) R-126	(2) R-128	(3) R-130	(4) R-132	(5) R-134	(6) R-136	(7) R-138	(8) R-140	(9) R-142	(10) R-144	(11) R-146	Average
					Sonic Soundings							
750	-8.3	-9.5	-9.5	-6.2	-6.2	-6.7	-6.6	-6.9	-6.5	-6.3	-6.0	7.02
1000	15.3	15.3	16.1	13.0	11.7	12.3	12.1	12.7	11.7	11.3	11.7	12.96
1250	21.8	21.3	21.2	18.7	18.7	18.3	18.3	17.8	17.8	17.5	17.8	18.95
1500	25.8	25.0	25.2	23.2	23.3	22.9	22.9	22.5	22.6	22.6	22.2	23.30
1750	28.3	28.3	28.4	26.9	25.9	26.8	26.8	26.4	26.2	26.0	26.2	27.00
2000	31.2	31.4	31.2	30.6	30.0	29.9	29.7	29.8	29.4	29.1	29.8	30.19
2250	33.3	33.4	33.1	32.6	32.5	32.3	32.4	32.3	31.9	31.7	32.1	32.49
2500	35.8	35.7	35.8	35.0	35.0	35.0	35.8	34.9	34.2	34.2	34.3	35.01
2750	38.0	38.0	38.3	37.3	37.1	37.2	37.4	37.3	37.0	36.4	36.5	37.30
3000	40.3	40.4	40.6	40.0	39.5	39.7	39.7	39.7	39.9	38.9	39.7	39.80
3250	42.3	42.6	42.9	41.9	42.0	42.1	42.6	42.1	42.0	41.9	42.0	42.27
3500	44.6	44.9	44.9	44.4	44.7	44.1	44.5	44.4	44.1	44.1	43.9	44.44
3750	46.8	47.0	47.4	46.8	46.6	46.7	46.9	46.7	46.7	46.6	46.7	46.75
4000	48.9	49.3	49.5	49.0	48.9	49.3	49.1	49.0	48.8	49.2	48.8	49.02
4250	51.5	51.7	52.4	51.2	51.2	51.8	51.1	51.6	50.9	51.3	51.0	51.41
					Leadline Soundings							
250	+0.1	+0.7	+0.6	+2.2	+2.0	+3.0	+1.7	-0.8	+0.2	+2.0	+1.1	+1.327
300	-0.9	-0.5	-0.4	-2.3	+1.9	+0.3	0.0	0.0	+0.5	-0.8	+0.5	+0.100
350	-1.5	-1.3	-1.6	-2.0	-0.1	+0.2	-0.2	-0.9	-0.3	-0.1	-0.8	-0.900
400	-2.3	-2.5	-3.0	-1.0	-1.9	-0.7	-0.8	-1.8	-1.5	0.0	-1.5	-1.182
450	-4.1	4.3	4.7	2.0	1.3	1.7	2.7	2.9	2.0	0.3	0.9	-2.325
500	-5.7	5.5	6.1	3.2	7.0	2.5	3.7	3.0	1.0	3.0	1.7	-3.455

TABLE 9

DEVIATION (in feet) OF ACTUAL PROFILE FROM AVERAGE PROFILE (12 May 1950)

PROFILE BEING COMPARED TO AVERAGE PROFILE

Distance from Base Line (ft.)	(1) R-126	(2) R-128	(3) R-130	(4) R-132	(5) R-134	(6) R-136	(7) R-138	(8) R-140	(9) R-142	(10) R-144	(11) R-146
					Sonic Soundings						
750	-1.28	-1.48	-2.48	+0.82	+0.82	+0.32	+0.42	+0.32	+0.32	+1.02	+1.02
1000	-2.34	-2.34	-3.14	-0.04	+1.26	+0.66	+0.86	+0.86	-1.26	+1.66	+1.26
1250	-2.85	-2.35	-2.25	+0.25	+0.25	+0.65	+0.65	+1.15	+1.15	+1.45	+1.15
1500	-1.80	-1.60	-1.80	+0.20	+0.10	+0.50	+0.50	+0.80	+0.80	+1.00	+1.20
1750	-1.30	-1.30	-1.10	+0.10	+0.20	+0.20	+0.20	+0.60	+0.80	+1.00	+0.80
2000	-1.02	-1.21	-1.01	-0.40	+0.19	+0.29	+0.49	+0.39	+0.79	+1.09	+0.39
2250	-0.81	-0.91	-0.61	-0.11	-0.01	+0.19	+0.09	-0.19	+0.59	+0.79	+0.39
2500	-0.79	-0.69	-0.79	+0.01	+0.01	+0.01	-0.79	+0.11	+0.81	+0.81	+0.71
2750	-0.70	-0.70	-1.00	0.00	+0.20	+0.10	-0.10	+0.00	+0.30	+0.90	+0.80
3000	-0.50	-0.60	-0.80	-0.20	+0.20	+0.10	+0.10	+0.10	-0.10	+0.90	+0.60
3250	+0.33	-0.43	-0.73	+0.27	+0.17	+0.07	+0.17	+0.07	+0.27	+0.27	+0.17
3500	-0.16	-0.46	-0.46	+0.04	-0.26	+0.14	+0.06	+0.04	+0.34	+0.34	+0.54
3750	-0.05	-0.25	-0.65	-0.05	+0.15	-0.15	-0.15	+0.05	-0.45	+0.15	+0.45
4000	+0.21	-0.29	-0.49	+0.01	+0.11	-0.29	-0.09	+0.01	-0.61	-0.19	+0.21
4250	-0.09	-0.29	-0.99	-0.21	+0.21	-0.39	+0.31	+0.01	+0.51	+0.11	+0.41
Total d	-13.00	-14.90	-18.60	+1.10	+3.60	+2.40	+3.30	+4.90	+9.70	+11.30	+10.20
Ave. d	-.8667	-.9933	-1.2400	+.0733	+.2400	+.1600	+.2200	+.3267	+.6467	+.7533	+.6800
					Leadline Soundings						
250	-1.23	-0.63	-0.73	+0.97	+0.67	+0.47	+0.87	-0.53	-0.43	+0.67	-0.23
300	-1.00	-0.60	-0.50	+1.90	+1.80	+0.20	-0.10	-0.10	+0.10	+0.70	+0.40
350	-0.70	-0.50	-0.80	-1.10	+0.70	+0.90	+0.70	-0.10	+0.70	+0.70	0.0
400	-0.92	-1.02	-1.52	+0.18	+0.28	+0.78	+0.68	-0.32	-0.02	+1.18	-0.02
450	+1.75	-1.95	-2.35	+0.35	+1.05	-0.55	+0.65	-0.45	-0.35	+2.05	+1.25
500	-1.95	-2.05	-2.65	+0.25	+1.45	+0.45	+0.25	+0.45	-0.45	+0.45	+1.75
Total d	-7.35	-6.75	-8.55	-0.45	+5.75	+4.15	+3.55	-1.05	+1.25	+8.05	+3.35
Ave. d	-1.22	-1.09	-1.42	-0.07	+0.96	+0.69	+0.59	-0.17	+0.21	+1.34	+0.56

TABLE 10

ERROR INTRODUCED BY USING GIVEN PROFILES ONLY, RATHER THAN ALL ELEVEN PROFILES (8000' Test Section Sonic Data)
(CUBIC FEET PER FOOT OF PROFILE PER FOOT OF BEACH)

Lines Number	6	1,11	1,9	1,6,11	3,6,9	1,...,8,11	2,5,7,10	1,4,6,8,11	1,3,6,9,11	2,4,6,8,10	1,3,5,7,9,11
Average Spacing (Feet)	2000	2000	1000	1000	700	600	500	500	500	400	400
12 May 1950	-.1600	+.0934	+0.2966	-.0333	+.1596	-.1120	-.0550	-.1040	+.1190	-.0640	+.0454
18 May	+.1840	-.1993	-.0660	-.0076	-.0050	+.0037	+.0506	+.0223	-.0277	+.0240	+.0153
26 May	+.1980	-.0020	-.0953	+.0080	+.0073	+.0400	-.0294	+.0680	+.0113	+.0047	+.0047
9 June	+.0827	-.0740	+.0700	+.0044	+.0790	-.0740	+.0160	+.0427	+.0480	+.0720	+.0673
16 June	+.0513	-.3387	+.0180	-.1236	+.0280	-.1150	+.0547	-.1223	-.0433	+.0167	+.0507
21 June	+.1547	-.2887	+.0987	-.0663	-.0227	-.0753	+.0380	-.0606	-.0606	+.0907	-.0340
23 June	+.2693	-.2240	-.0964	+.0227	+.0992	-.0210	+.0110	+.0197	+.0492	+.0653	-.0105
30 June	+.2233	-.1700	+.0333	-.2283	+.0903	-.2440	+.0100	-.0013	+.1197	+.0353	+.0007
7 July	-.0700	+.0800	-.0767	+.0450	-.0257	-.0793	+.0100	+.0273	+.0044	+.0260	-.0893
21 July	+.2420	-.2780	+.0220	-.0180	+.0886	-.0240	+.0270	+.0120	+.0280	+.0047	+.0073
4 August	+.0287	-.0698	+.0287	-.0255	+.0257	+.0132	+.0037	+.0071	+.0059	-.0160	+.0395
11 August	+.2620	-.1723	+.0254	+.0653	+.0823	-.0453	+.0087	+.0053	+.0080	-.0233	+.0550
18 August	+.0993	-.2107	+.1727	-.0556	+.1507	-.0263	-.0323	-.0170	+.0740	-.0727	+.1117
25 August	+.0353	-.1480	+.1220	-.0513	-.0960	-.0263	-.0087	+.0246	+.0452	+.0153	+.0127
1 September	+.1153	-.2414	+.0797	-.0631	+.0897	-.0650	+.0470	-.0443	+.0257	+.0033	+.0453
8 September	+.2073	-.1074	+.0000	-.0489	+.0527	-.0113	-.0127	+.0240	+.0406	-.0167	+.0063
24 April 1951	-.2747	-.4353	-.1180	-.3052	+.1600	-.0410	+.0380	-.0217	+.0393	+.0073	+.0393
27 April	-.0870	-.3596	+.1473	-.1043	+.0963	-.0476	+.0190	-.0584	+.0263	+.0120	+.0233
28 April	-.1680	-.1066	-.0497	+.0317	+.0163	-.0807	+.0713	-.0354	+.0044	-.0414	+.0633
Standard Deviation	.169	.205	.107	.079	.086	.060	.034	.056	.058	.045	.047
Probable error	.114	.139	.072	.053	.058	.041	.023	.038	.039	.030	.032

TABLE 11

ERROR INTRODUCED BY USING GIVEN PROFILES ONLY RATHER THAN ALL 47 PROFILES (9200' Section-Sonic Data)
(CUBIC FEET PER FOOT OF PROFILE PER FOOT OF BEACH)

Line Number	Average Spacing (Feet)	Jun 1949	Oct 1949	Feb 1950	Apr 1950	Jun 1950	Sept 1950	Dec 1950	Apr 1951	Standard Deviation (Feet)
1,3,5,7,...47	400	+0.0904	+0.0274	+0.0513	+0.0565	+0.0385	+0.0417	+0.0521	+0.0633	.0568
2,4,6,8,...46	400	+0.0033	+0.0743	+0.0759	+0.0491	+0.0277	+0.0466	+0.0492	+0.0306	.0519
2,5,8,11,...44	600	+0.1052	+0.0462	+0.0072	+0.0718	+0.0707	+0.0473	+0.0700	+0.0089	.0615
1,4,7,10,...46	500	+0.0485	+0.0632	+0.1007	+0.0350	+0.0507	-0.0377	+0.0054	+0.0726	.0581
3,6,9,12,...45	600	+0.0012	+0.0488	+0.0659	+0.0649	+0.0904	+0.0529	+0.0127	+0.0551	.0539
1,4,7,...22,26,29,32,...47	600	-0.0762	-0.1004	-0.1023	-0.1051	-0.1087	-0.0750	-0.0579	+0.1197	.0952
1,5,9,13,...45	800	+0.0204	-0.0306	+0.0158	+0.0552	+0.0541	-0.0087	+0.0203	+0.1556	.0637
2,6,10,14,...46	800	-0.0560	-0.0796	+0.0243	-0.0006	+0.0910	+0.0239	+0.0449	+0.1006	.0682
3,7,11,15,...47	800	+0.1723	+0.0853	+0.0681	+0.2578	+0.4230	+0.0747	+0.0806	+0.0325	.0860
1,5,9,...21,24,27,31,35,...47	800	+0.1698	-0.0032	+0.2006	+0.1002	+0.0552	+0.0952	-0.0531	+0.0365	.0903
1,6,11,16,...46	1000	+0.0303	+0.0006	+0.0133	-0.0649	+0.2815	-0.0057	-0.0600	+0.0109	.0502
2,7,12,17,...47	1000	-0.1041	-0.0247	+0.0282	+0.0877	+0.1690	+0.0300	-0.0414	+0.3457	.0930
1,6,11,16,21,27,32,37,42,47	1000	+0.3151	-0.0595	-0.0176	+0.0112	+0.0444	+0.0177	-0.0871	+0.0381	.120
4,9,14,...44	1000	+0.1332	+0.0533	+0.0581	+0.0292	-0.0448	+0.0487	+0.0949	+0.0403	.104
1,7,13,19,24,29,35,41,47	1100	+0.0944	-0.0202	+0.0317	+0.0032	+0.0196	+0.1253	-0.0252	-0.0553	.0611
4,10,16,22,25,32,38,44	1200	+0.0769	+0.3079	+0.0981	+0.1400	+0.0713	+0.0232	+0.0801	+0.0520	.0876
4,10,16,21,26,32,38,44	1200	-0.0031	+0.1066	-0.0924	+0.2138	+0.0801	+0.0211	+0.0887	+0.0001	.103
4,10,16,22,27,32,38,44	1200	-0.0745	+0.0904	-0.0775	+0.1058	+0.0757	+0.0408	+0.0445	+0.1039	.0794
1,8,15,21,27,33,40,47	1300	+0.0577	+0.0883	+0.1244	+0.0884	+0.1297	+0.1011	+0.0032	+0.1339	.0996
3,10,17,24,31,38,44	1350	-0.1293	+0.0127	+0.0313	+0.1008	+0.1135	+0.2922	+0.1286	+0.0313	.0922
1,8,16,24,32,40,47	1550	-0.0426	+0.0229	-0.0397	+0.1907	+0.1198	+0.1029	-0.0023	-0.0009	.0737
5,13,21,28,35,43	1600	-0.0947	-0.0134	+0.2704	+0.1004	-0.0772	+0.0796	+0.0716	-0.0744	.108
5,13,20,27,35,43	1600	+0.0302	+0.0033	+0.2401	+0.1797	-0.0602	+0.2006	+0.1258	+0.1380	.146
1,10,19,29,38,47	1800	+0.0706	+0.1106	-0.1384	-0.0378	+0.1524	+0.0662	-0.3023	+0.1174	.145
6,15,24,33,42	1800	-0.0037	-0.0478	+0.0452	+0.1448	+0.1299	+0.1656	+0.1933	+0.0224	.118
1,12,24,36,47	2300	-0.2072	-0.0492	+0.3793	-0.1527	+0.1822	+0.0045	-0.2654	+0.2633	.185
1,13,24,35,47	2300	-0.2162	-0.0548	+0.2681	-0.0143	+0.0278	+0.1328	-0.1233	-0.3937	.194
7,18,30,41	2300	+0.0303	+0.0287	-0.0543	+0.0337	+0.0483	+0.1787	+0.3220	+0.2090	.153
1,16,32,47	3100	-0.3222	-0.2543	+0.0763	+0.0092	+0.1716	-0.0434	-0.2871	+0.1786	.197
9,24,39	3100	+0.1806	+0.3031	+0.2817	+0.3678	+0.4065	+0.1808	+0.0499	+0.1995	.312
1,24,47	4600	-0.2780	-0.4947	-0.7433	-0.3263	+0.0833	-0.0700	+0.2187	+0.5777	.415
13,38	4600	+0.3387	+0.1463	+0.6673	+0.3120	+0.1900	+0.1880	+0.2890	+0.1127	.396
12,36	4600	-0.3233	+0.3287	+0.3373	+0.1547	+0.2867	+0.0720	-0.1647	-0.0390	.397
1,47	9200	-2.3060	-2.3380	-2.2993	-2.1447	-1.9867	-2.0247	-2.3313	-2.1193	2.236
24	9200	+1.7520	+1.3087	+2.1507	+1.7920	+1.9902	+1.7353	+1.4353	+0.9640	1.680

TABLE 12

ERROR INTRODUCED BY USING GIVEN PROFILES ONLY, RATHER THAN ALL ELEVEN PROFILES (2000' Test Section-Lead-Line Data)
(CUBIC FEET PER FOOT OF PROFILE PER FOOT OF BEACH)

Lines Number	6	1,11	3,9	1,6,11	3,6,9	1,4,8,11	2,5,7,10	1,4,6,8,11	1,3,6,9,11	2,4,6,8,10	1,3,5,7,9,11
Average Spacing (feet)	2000	2000	1000	1000	700	600	500	500	500	500	400
12 May 1950	-.6917	+.3333	+.6083	+.1791	-.0958	+.1875	-.3583	+.0942	+.1634	-.0650	+.0600
18 May	-.1633	-.1517	+.2050	-.1825	-.0795	+.1050	-.2242	-.1159	+.0092	-.0300	+.0617
26 May	-.5450	+.1633	-.1533	-.1908	-.2708	+.3500	-.0025	+.9300	-.2075	-.0250	-.0067
9 June	-.1983	+.0267	+.3017	-.0859	+.1517	+.2500	-.1142	+.0017	+.0967	-.0458	+.0400
16 June	-.0920	-.0400	-.0587	-.0650	-.0667	-.0625	+.0017	+.0375	-.0633	+.0567	-.0857
21 June	-.1433	+.0233	+.0900	-.0606	+.0202	+.0700	-.0642	+.0233	+.0067	+.0233	-.0267
23 June	+.2457	+.0943	-.2117	+.0675	-.1342	+.1000	-.0033	+.0683	-.0742	-.0233	+.0417
30 June	-.1703	-.1383	+.1033	+.2200	+.1258	-.0150	-.0258	-.0083	+.0775	+.0483	+.0183
7 July	+.0533	-.0427	-.1250	+.0083	+.0202	-.0038	+.0583	+.0075	-.0333	+.0117	-.0050
21 July	-.0773	-.2477	-.0607	-.0149	-.0567	+.0902	+.0143	+.0108	-.0415	+.0727	-.0623
4 August	-.2530	-.0947	+.1053	+.1238	+.0279	-.2088	+.0762	-.1022	-.0122	-.0153	+.0353
12 August	-.2697	-.1947	-.2647	-.2321	-.2522	+.2237	+.0425	+.1320	-.2488	+.1670	-.1250
18 August	-.0898	+.0455	+.0705	-.0712	+.0238	+.1417	-.1275	+.0897	+.0180	+.2095	-.0415
25 August	+.2157	+.1667	+.0267	+.1417	+.0467	-.0375	-.0523	+.0108	+.0167	-.0300	-.0033
1 September	+.1595	+.0652	-.1025	+.0568	-.0565	-.1690	+.0653	-.0345	-.0732	-.0448	+.0318
8 September	-.1485	+.0182	+.2266	-.0652	+.1110	-.1802	+.0473	-.1568	+.0723	-.0352	+.0315
24 April 1951	-.0617	+.2613	-.2637	+.0988	-.2037	+.0595	+.1155	-.0328	-.0967	-.2770	+.1247
27 April	+.6622	+.2705	-.3378	+.4163	-.0378	-.0512	-.0067	-.1276	+.0638	+.1088	-.1478
28 April	+.4773	+.1448	-.4227	+.3107	-.1527	-.0427	+.0815	+.0773	-.0383	+.1107	-.1393
Standard Deviation	.301	.145	.242	.162	.127	.151	.119	.226	.099	.077	.074
Probable Error	.201	.098	.163	.109	.086	.102	.080	.152	.067	.052	.050

TABLE 13

ERROR INTRODUCED BY USING GIVEN PROFILES ONLY RATHER THAN ALL 47 PROFILES (7200' Section-Lead-Line Data)
(CUBIC FEET PER FOOT OF PROFILE PER FOOT OF BEACH)

Lines Number	Average Spacing (feet)	Jun 1949	Oct 1949	Feb 1950	Apr 1950	June 1950	Sep 1950	Dec 1950	Apr 1951	Standard Deviation (foot)
1,3,5,7,...47	600	-0.0149	+0.1062	+0.0010	+0.0167	+0.0909	+0.0599	+0.0333	+0.0758	.0527
2,4,6,8,...46	600	+0.0764	-0.0237	+0.2724	+0.0792	+0.0504	+0.0102	+0.0167	-0.0806	.0515
2,5,8,11,...47	600	+0.0112	+0.0384	+0.0030	+0.0308	-0.0046	-0.0179	+0.0237	+0.0954	.0396
1,4,7,10,...46	600	-0.0641	+0.0149	+0.1006	-0.0950	-0.0127	-0.0236	+0.0592	+0.1350	.0761
3,6,9,12,...45	600	+0.1645	+0.0808	+0.0077	+0.2046	+0.1521	+0.1316	+0.0045	-0.1552	.1322
1,4,7,...22,25,29,32...47	600	-0.2250	-0.0783	-0.0456	-0.2129	-0.1367	-0.2702	-0.2957	+0.0380	.1325
1,5,9,13,...45	800	-0.0909	+0.0268	+0.0192	+0.0299	+0.0664	0.1102	+0.0354	-0.0293	.0619
2,6,10,14,...46	800	+0.0040	-0.2764	+0.0677	+0.1854	+0.0372	+0.0621	+0.0439	+0.0966	.0858
3,7,11,15,...47	800	+0.0617	+0.1551	-0.0370	+0.0266	-0.0849	+0.0015	+0.0362	+0.1509	.0898
1,5,9,...21,24,27,31,35,...47	800	-0.1190	+0.1928	+0.2292	-0.0486	+0.0125	-0.1714	-0.0687	+0.1102	.1703
1,6,11,16,...46	1000	-0.1895	+0.0594	+0.0813	+0.0759	-0.0463	+0.0057	+0.0833	+0.1289	.2071
2,7,12,17,...47	1000	-0.0888	-0.1132	-0.0106	-0.1606	-0.0633	-0.0723	-0.0339	-0.0806	.0968
1,6,11,16,21,27,32,37,42,47	1000	-0.2384	+0.7130	-0.2037	+0.0419	-0.0618	-0.0017	-0.0589	-0.0300	.1058
4,9,14,...44	1000	+0.1968	+0.0255	-0.0053	+0.0729	+0.0874	+0.2350	-0.0149	-0.0201	.1150
1,7,13,19,24,29,35,41,47	1200	-0.3674	-0.0626	+0.0786	-0.2023	+0.0934	+0.1367	-0.0618	-0.0378	.1534
4,10,16,22,26,32,38,44	1200	+0.0457	+0.1470	+0.1085	-0.0109	-0.0119	+0.0247	+0.0109	+0.1649	.0884
4,10,16,21,26,32,38,44	1200	+0.0787	+0.2719	+0.1321	+0.0232	+0.0423	+0.0612	+0.0080	+0.0759	.1115
4,10,16,22,27,32,38,44	1200	+0.0821	+0.1697	+0.0115	-0.0357	+0.0080	+0.0454	-0.0259	+0.3682	.0929
1,8,15,21,27,33,40,47	1300	-0.0614	+0.1540	-0.0734	-0.0032	+0.0671	+0.0577	+0.0386	-0.1685	.0896
4,10,17,24,31,38,44	1350	+0.0607	+0.1649	+0.1201	-0.1266	+0.1214	+0.0663	+0.0011	+0.0999	.1077
1,8,16,24,32,40,47	1550	-0.2944	+0.0875	+0.2636	+0.1598	+0.1448	-0.0862	+0.0078	-0.2391	.1829
5,13,21,28,35,43	1600	-0.0088	+0.1377	+0.1101	-0.0219	+0.1129	+0.2006	-0.0792	-0.1248	.1651
5,13,20,27,35,43	1600	-0.0654	+0.1047	+0.1195	+0.0517	+0.0890	+0.1441	-0.0739	+0.4286	.1709
1,10,19,29,38,47	1800	-0.3014	+0.1350	-0.2376	-0.1089	-0.0475	+0.0134	-0.2859	-0.1731	.1896
6,15,24,33,42	1800	-0.4045	+0.0788	-0.0073	+0.2258	+0.1329	+0.1636	+0.1091	-0.3242	.2629
1,12,24,35,47	2300	-0.2786	-0.4466	+0.0939	-0.3259	+0.0501	+0.1477	-0.2598	-0.2017	.2503
1,13,24,35,47	2300	-0.3880	-0.1618	+0.3036	-0.4583	+0.0941	+0.2273	-0.1527	+0.1336	.2684
7,18,30,41	2300	-0.2167	+0.0667	+0.0717	+0.1850	+0.1883	-0.0363	+0.3375	-0.5163	.2447
1,16,32,47	3100	-0.5696	+0.2232	+0.0242	-0.4331	+0.1829	+0.1264	+0.0011	-0.2412	.3369
9,24,39	3100	-0.0620	+0.2006	+0.4401	+0.0457	+0.1518	+0.7275	-0.0701	+0.2316	.3583
1,24,47	4600	-0.3416	-0.8708	-0.1992	-0.6234	+0.1033	+0.1200	-0.4530	-0.5183	.5409
11,35	4600	+0.0917	+0.5917	+0.5717	-0.2400	+0.1617	+0.3867	+0.1750	+0.8367	.5102
12,36	4600	+0.2583	-0.0667	+0.1717	-0.0617	-0.0800	+0.1033	-0.1000	+0.1033	.2655
1,47	7200	-1.4417	-1.8917	-1.7033	-1.8483	-1.6633	-1.5383	-1.1500	-1.2633	1.5283
24	9200	-0.2417	+0.1500	+1.3050	+0.6017	+1.0700	+1.7783	+0.2508	+0.1866	1.0549

TABLE 14

Average Spacing Error

Spacing	Sonic Standard deviation (feet)	Sonic Probable error (feet)	Leadline Standard deviation (feet)	Leadline Probable error (feet)
2000-foot Test Section				
2000	0.188	0.127	0.236	0.159
1000	0.094	0.064	0.206	0.139
650	0.074	0.050	0.140	0.094
500	0.051	0.034	0.158	0.107
400	0.046	0.031	0.0751	0.051
9200-foot Full Section				
9200	1.977	1.333	1.345	0.907
4600	0.399	0.269	0.443	0.299
3100	0.260	0.175	0.348	0.235
2300	0.178	0.120	0.257	0.173
1800	0.132	0.089	0.229	0.155
1525	0.113	0.076	0.178	0.120
1300	0.096	0.065	0.099	0.067
1200	0.084	0.057	0.118	0.079
1000	0.095	0.064	0.102	0.069
800	0.077	0.052	0.110	0.074
600	0.069	0.047	0.103	0.069
400	0.054	0.037	0.052	0.035